Physics and QIP in Maynooth

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Maynooth University
National University of Ireland Maynooth

EQUITY Workshop, UCD, May 2022

Science Foundation Ireland sfi For what's next

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Something about me...

QIP Expertise + Inspiration: Topological Quantum Computation

Related physics expertise

- Anyons.
    * Theory (TQFT/anyon models)
    * 2D Systems/models with anyons
    * Detection/devices - anyon interferometers

- Quantum Hall Physics
    * Trial wave functions
    * Edge/entanglement excitation spectra (CFT),
    * Point contacts/interferometers

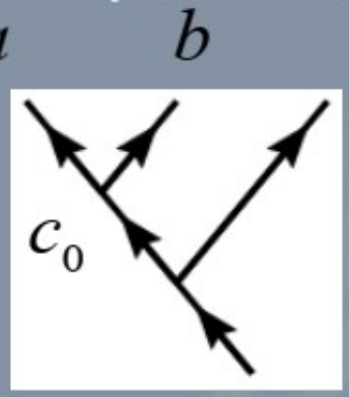
- Theory of topological phase transitions
    * Anyon condensation
    * Quenches

- Majorana wires and generalizations
    * Error processes, resonances
    * Graph braiding

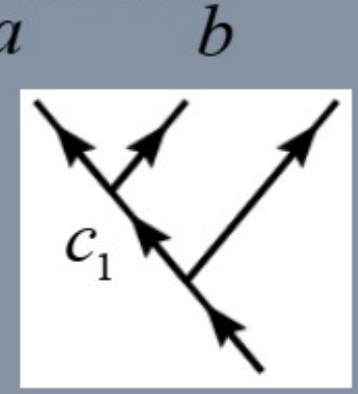
Anyonic Quantum Computation

Qubits formed with 3 anyons. Information in fusion channel of the 2 on the left

$|0\rangle \rightarrow$

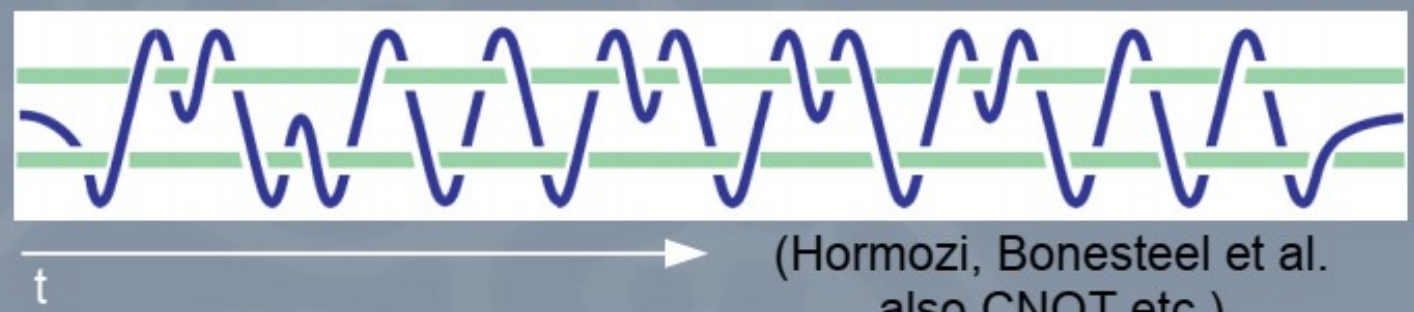


$|1\rangle \rightarrow$

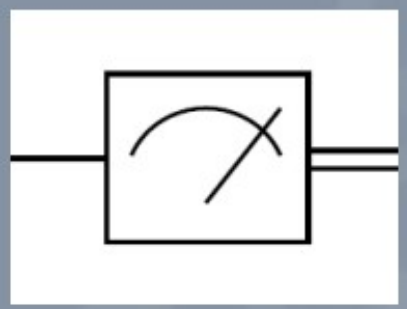


Quantum gates implemented by "braiding" the anyons

"QNOT" $\rightarrow$



(Hormozi, Bonesteel et al.
also CNOT etc.)



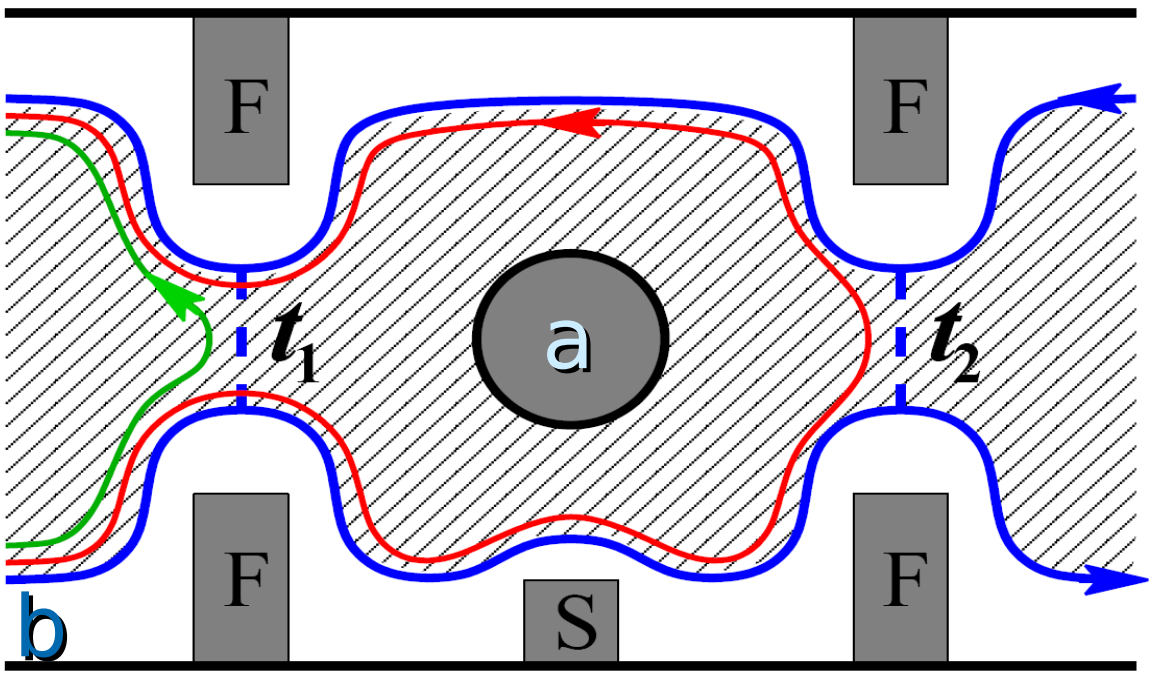
Readout requires measurement of non-local topological charges

$\rightarrow$ Anyon Interferometer

(more on this soon)

Note: the anyonic quantum computation scheme is equivalent to the circuit model (they can simulate each other effficiently)

Quantum Hall Interferometry – Detecting non-Abelian Anyons



Top view of Hall interferometer

Type a anyon under top gate

Currents runs along the edges,
Tunneling at constrictions (F)
(Type b anyons tunnel)

Area varied with side gate S

$$\sigma_{xx} \propto |t_1|^2 + |t_2|^2 + 2\mathrm{Re}\left\{t_1^* t_2 \langle \Psi_{ab}| U_1^{-1} U_2 |\Psi_{ab}\rangle\right\}$$
$$= |t_1|^2 + |t_2|^2 + 2|t_1 t_2||M_{ab}|\cos(\beta + \theta_{ab}) \ .$$

Tunneling amplitudes t1,t2, unitaries U1,U2 applied to red/green paths,
Geometric+Aharonov-Bohm phase β varied with gate S; gives interference pattern,
Θ and M are effects from braiding of anyons of type a and b.

Interference suppressed by |M|: effect from non-Abelian braiding!

(Can even have |M|=0. Then see no interference for some topological charges a! Expected at ν=5/2)

J. Nakamura, S. Liang, G. C. Gardner & M. J. Manfra,
Nature Physics 16, 931 (2020); Preprint arXiv:2006.14115

BREAKING NEWS!!!
Abelian braiding detected!

Colour plot of measured interference current's phase, expected: $\theta = 2\pi \frac{e^*}{e} \frac{A_I B}{\Phi_0} + N_{qp}\theta_{anyon}$

Here, θ_{anyon}=1/3

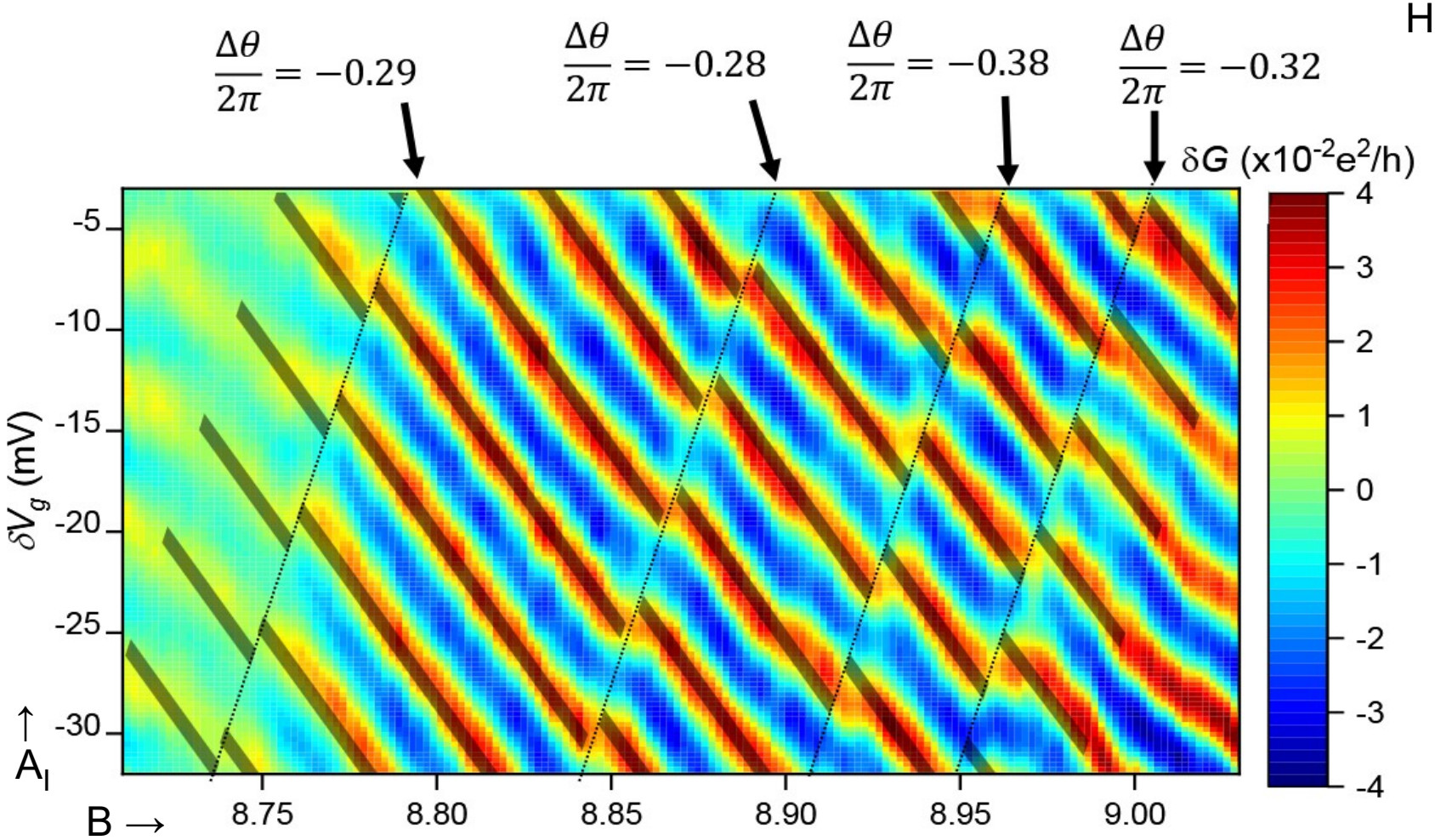
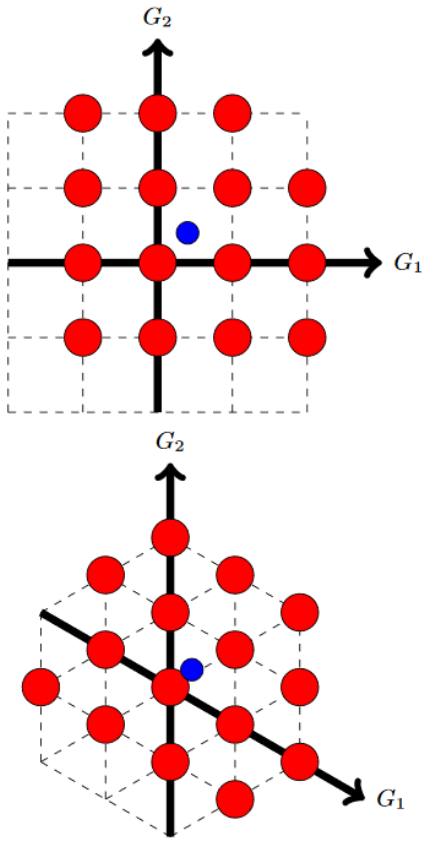


FIG. 2. **Conductance oscillations versus magnetic field and side gate voltage.** The predominant behavior is negatively sloped Aharonov-Bohm interference, but a small number of discrete phase jumps are visible. Dashed lines are guides to the eye for these features. Least-squares fits of $\delta G = \delta G_0 \cos\left(2\pi \frac{AB}{\Phi_0} + \theta_0\right)$ are shown with highlighted stripes, and the extracted change in phase $\frac{\Delta\theta}{2\pi}$ are indicated for each discrete jump. Increasing magnetic field is expected to reduce the number of localized quasiparticles; therefore the change in phase across each jump is predicted to be $-\theta_{anyon}$.

M.Fremling, N.Moran, JKS and S.H.Simon
Trial wave functions for a Composite Fermi liquid on a torus
Phys.Rev.B 97, 035149, 2018

More recently...
Shining our light on
Composite Fermions

$$\Psi_{\text{trial}}(\mathbf{z}) = P_{\text{LLL}}\left[\psi_{CF}(\mathbf{z})\left(\prod_{i<j}(z_i - z_j)^{2m}\right)e^{-\frac{1}{4}\sum_j |z_j|^2}\right]$$



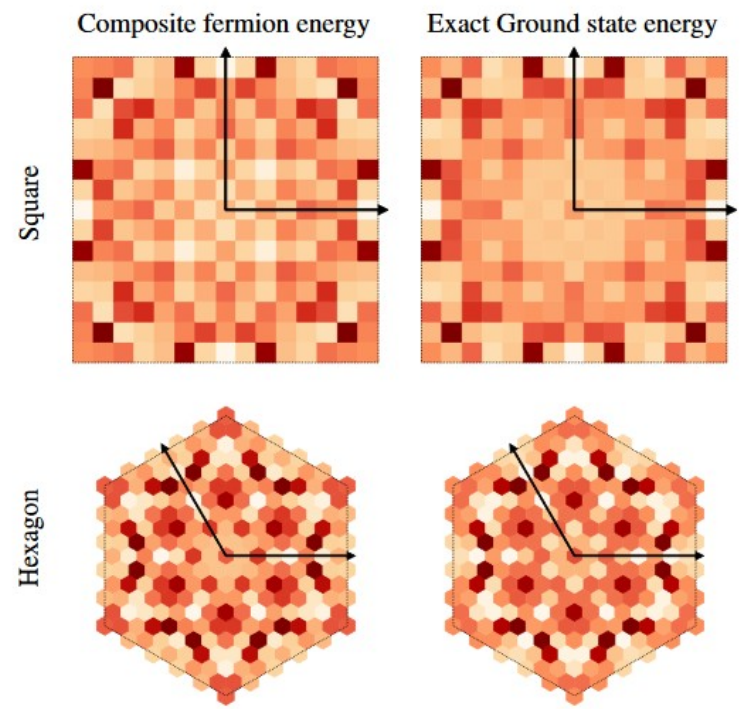
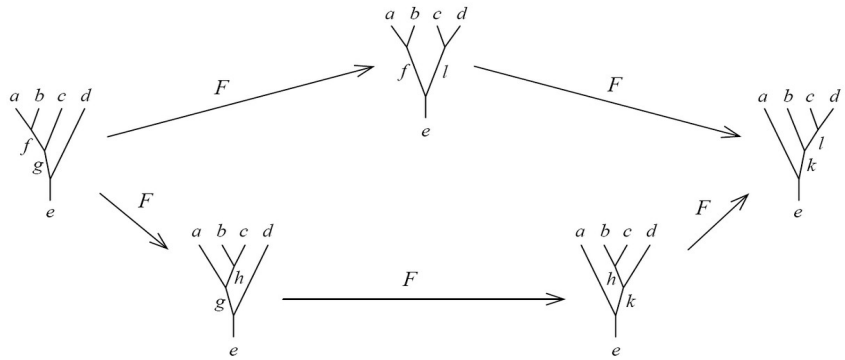


FIG. 1. Comparison between exact energy landscape and composite fermion energy landscape, given by (4) for $N_e = 15$. The lowest (highest) energy is at the darkest (lightest) red. The ground states at

Fermi balls at 15 particles

Torus energy landscapes compared
Overlaps are also very good
(for CF groundstates – dark points)

Discovering Anyon Models by Computer

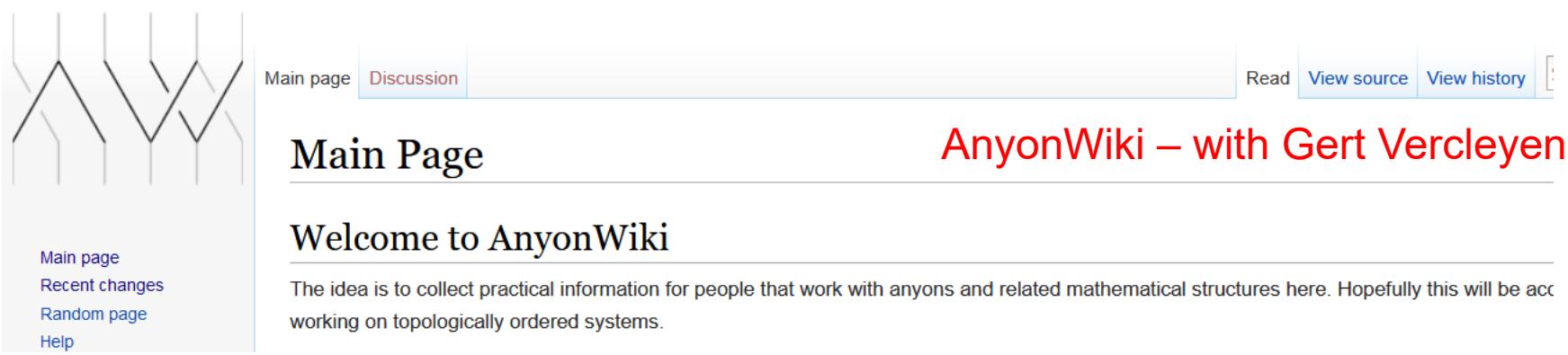


Consistency:
Pentagon/Hexagon equations

Finite number of anyon models
With N topological charges

1			A=abelian N=non-abelian U=universal for anyonic QC #=number of UNITARY theories	
A 2 **$SU(2)_1$** Semion=Z_2	N 2 **$SO(3)_3$** Fib U2			
A 2 **Z_3** (ν=1/3)	N 8 **$SU(2)_2$** **Ising** (ν=5/2)	N 2 **$SO(3)_5$** U4		
A 4 **Z_4**	N 4 **$SU(2)_3$** (ν=12/5) U	N 2 **$SO(3)_7$** U6	N 4 **Fib×Fib** U	A 6 **$Z_2 \times Z_2$**

Periodic table of modular anyon models (Z. Wang)



Aaron Conlon, Domenico Pellegrino, JKS, Shane Dooley, Graham Kells
Error generation and propagation in Majorana-based topological qubits
Phys. Rev. B 100, 134307, 2019

Quantum Wires

Error generation and propagation in Majorana-based topological qubits

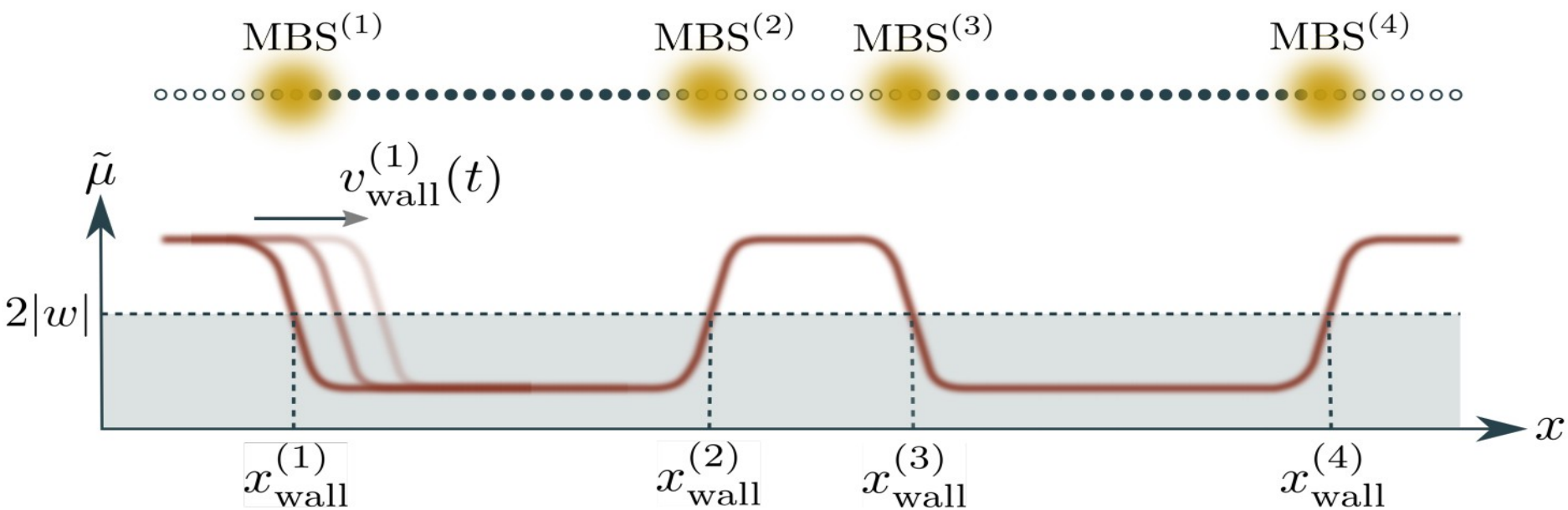
A. Conlon,[1,*] D. Pellegrino,[1] J. K. Slingerland,[1,2] S. Dooley,[2] and G. Kells[2]

[1]*Department of Theoretical Physics, Maynooth University, Ireland.*

[2]*Dublin Institute for Advanced Studies, School of Theoretical Physics, 10 Burlington Rd, Dublin, Ireland.*

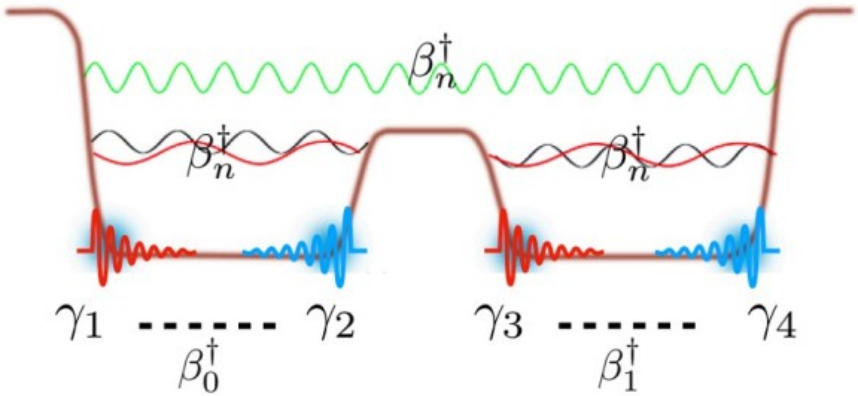
(Dated: May 17, 2019)

We investigate dynamical evolution of a topological memory that consists of two p-wave superconducting wires separated by a non-topological junction, focusing on the primary errors (i.e., qubit-loss) and secondary

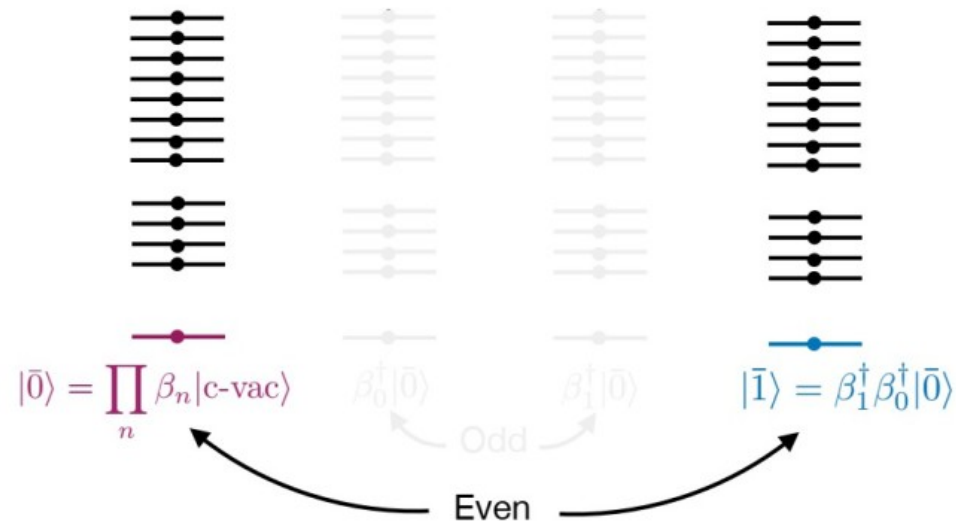


A simple Majorana-based memory

quasi-particle picture

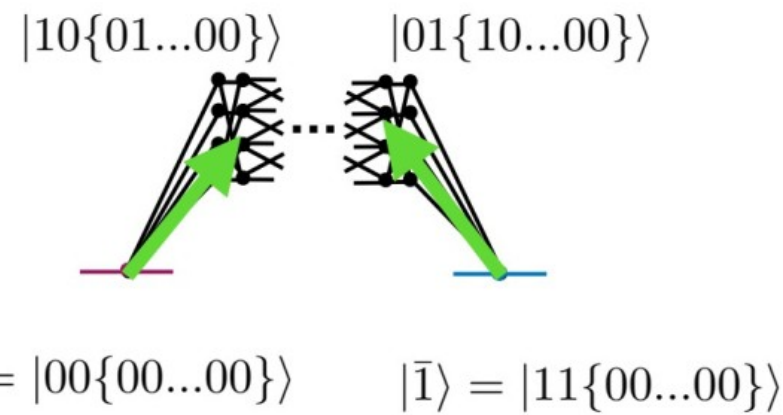


eigenstate picture



Dynamics under time-dependent perturbations

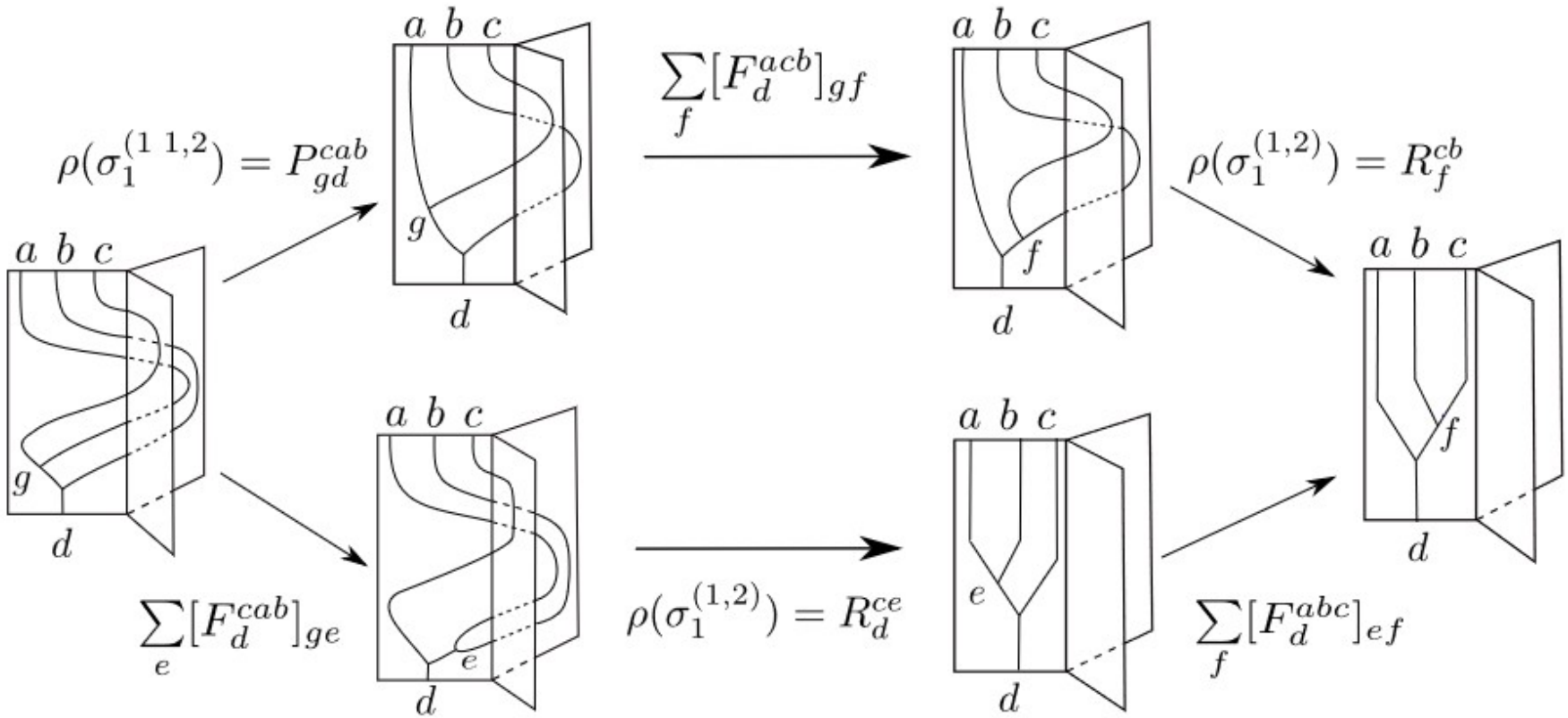
1. Qubit Loss
2. Bit-Flip error
3. Phase-Flip error



Karzig, Refael, von Oppen, PRX 3, 041017 (2013)
Scheurer and Schnirman, PRB 88, 064515 (2013)

Fusion and Braiding on Graphs / Wire Networks

Compatibility of Braiding and Fusion on Wire Networks
Aaron Conlon, JKS
arXiv:2202.08207



One of the graph hexagon equations.

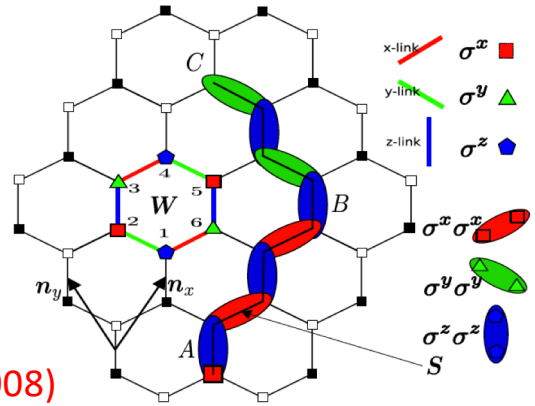
(different from the planar hexagons – new types of anyons!)

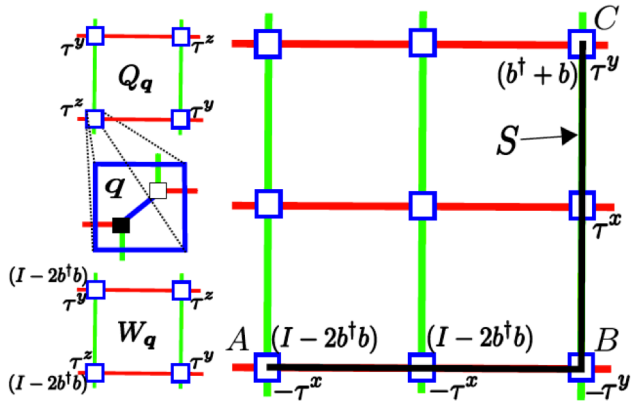
Topological Quantum Computation: topological phases

Kitaev honeycomb spin-1/2 lattice system

The phase diagram exhibits

- Abelian toric code phase
- non-Abelian Ising topological phase





Our solution using

- Jordan-Wigner fermionization
- Bogoliubov-de Gennes formalism

Kells, et al., Phys. Rev. Lett. 101, 240404 (2008)

Kells et al., Phys. Rev. B 80, 125415 (2009)

Generalizations: Yao-Kivelson and our Square-Octagon models

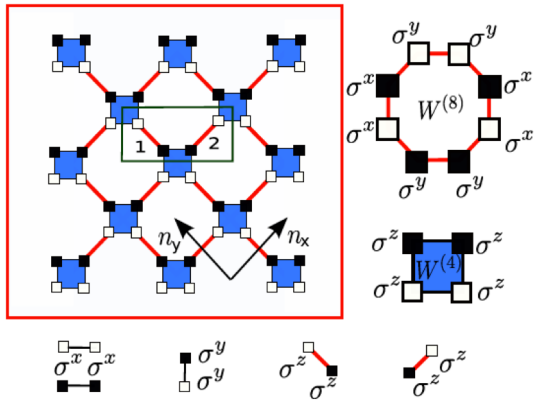
The phase diagram of Square-Octagon model has two non-Abelian phases

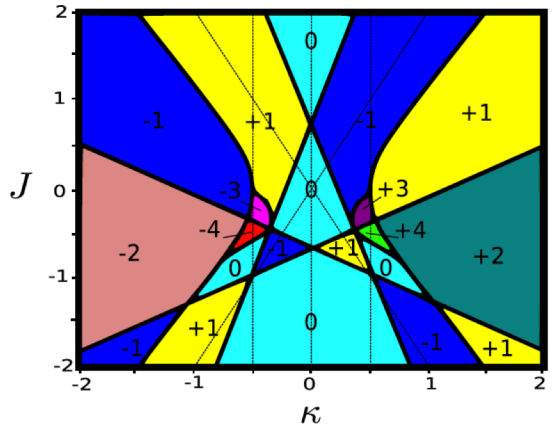
- Ising topological phase $\nu=\pm1$
- $SU(2)_2$ phase $\nu=\pm3$

Kells et al., Phys. Rev. B 81, 1104429 (2010)

Kells et al., New J. Phys. 13, 095014 (2011)

Kells et al., Phys. Rev. B 89, 075122 (2014)

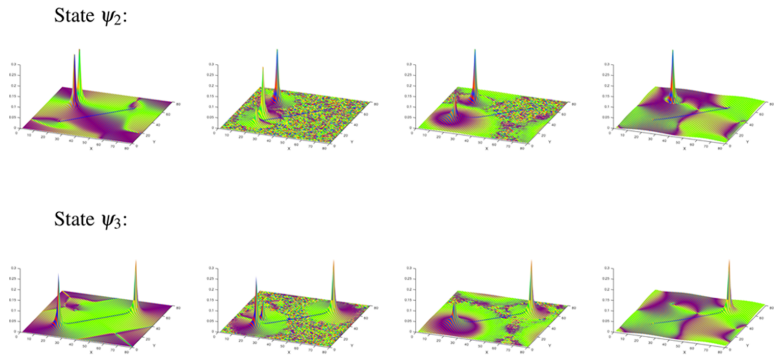




Topological Quantum Computation: defects and Majorana fermions

Dynamics of Majorana bound states

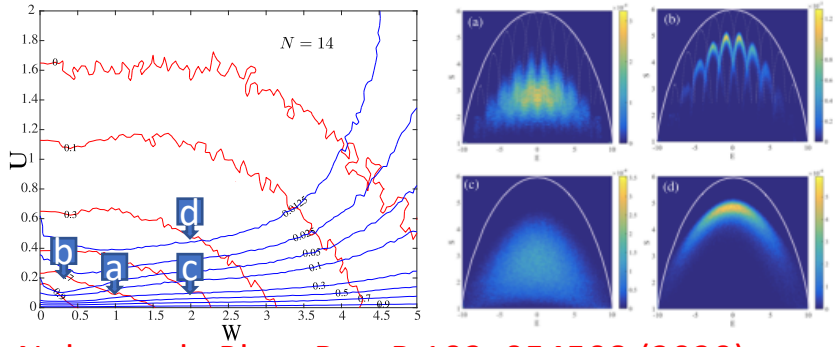
- annihilation of Majorana states bound to mobile vortices and static line defect end-points



Brennan, Vala, J. Phys. Chem. A **120**, 3326 (2016)
Bolukbasi, Vala, New J. Phys. 14, 045007 (2012)

Constrained thermalization and topological superconductivity

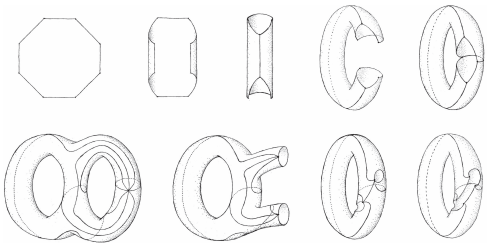
- Majorana chains with interactions U and disorder W
- numerical phase diagram and entanglement entropy

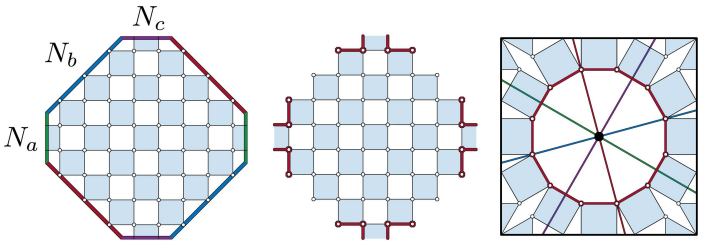


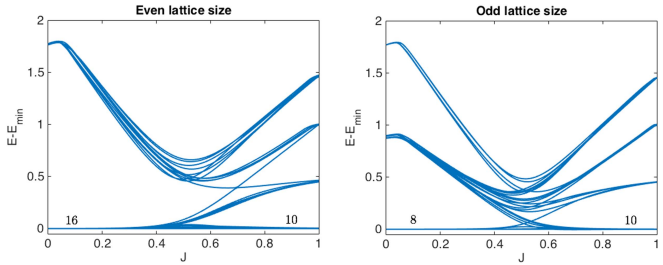
Nulty et al., Phys. Rev. B 102, 054508 (2020)

Generalization of the Kitaev model on higher genus surfaces

- lattice on a surface of an arbitrary genus is composed from lattices on genus 2 surfaces that tesselate an octagon
- the ground state degeneracy depends on genus and even/odd lattice size and changes at the phase transition







Brennan, Vala, New J. Phys. 20, 053023 (2018)

Quantum Computing Gates and Circuits

Cartan decomposition of two-qubit operations

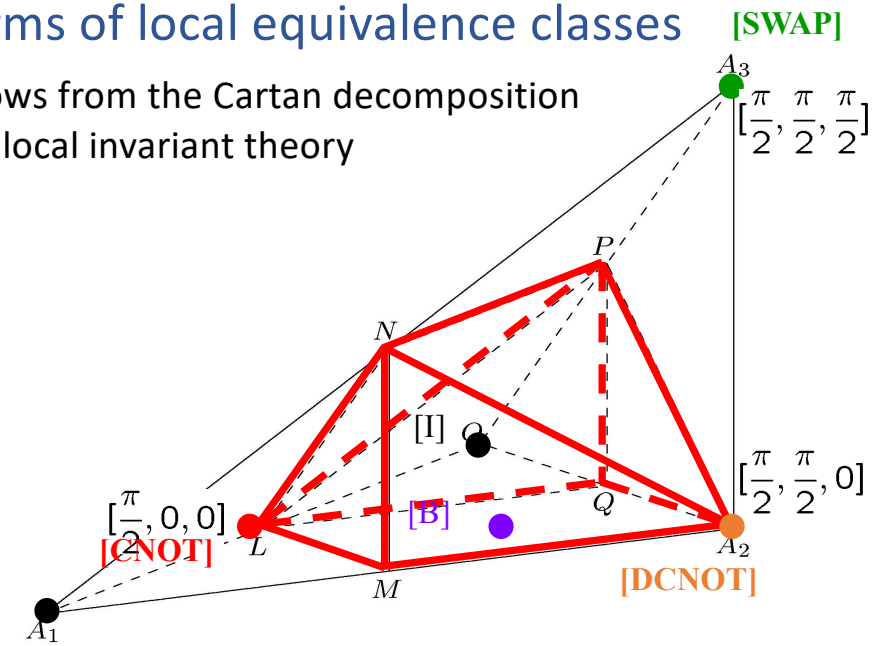
$$U = k_1 \, A \, k_2 = k_1 \, e^{\frac{i}{2}\left(c_1 \, \sigma_x^1 \sigma_x^2 + c_2 \, \sigma_y^1 \sigma_y^2 + c_3 \, \sigma_z^1 \sigma_z^2 \right)} \, k_2 \qquad k_1, \, k_2 \in SU(2) \otimes SU(2)$$

Local equivalence

- two operations are locally equivalent if they have the same **entangling power**
- follows from the SU(4) Cartan decomposition and local invariance theory

Classification of two-qubit operations in terms of local equivalence classes

- follows from the Cartan decomposition and local invariant theory



Polyhedron of perfect entanglers

Definition

A two qubit gate is called a **perfect entangler** if it can produce a maximally entangled state from a product state.

How large portion of all two-qubit gates are perfect entanglers?

$$V(\mathcal{U}) = \int_{(SU(2)\otimes SU(2))\times(SU(2)\otimes SU(2))\times\mathcal{U}} \mathrm{d}\mu = \int_{\mathcal{U}} \mathrm{d}\mu_{\mathcal{A}}$$

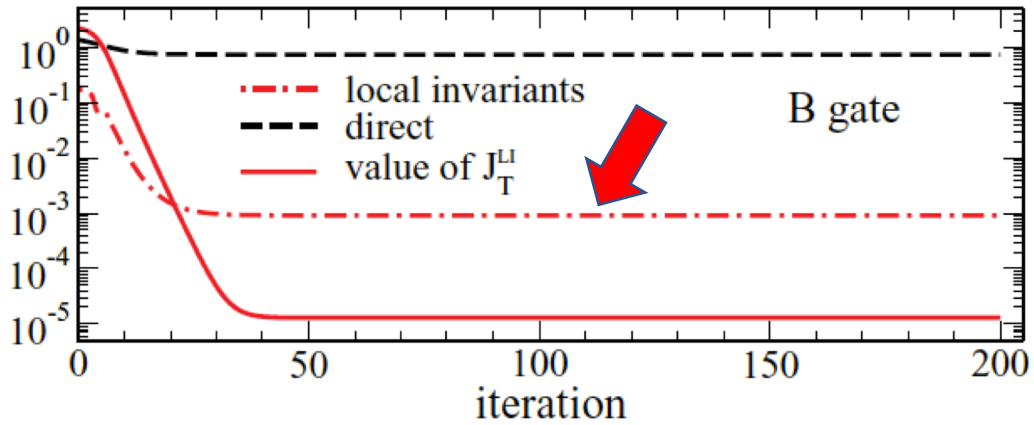
Watts, Vala, Entropy 15, 1963–1984 (2013)

Quantum Control

Quantum control towards a target given by an entire local equivalence class

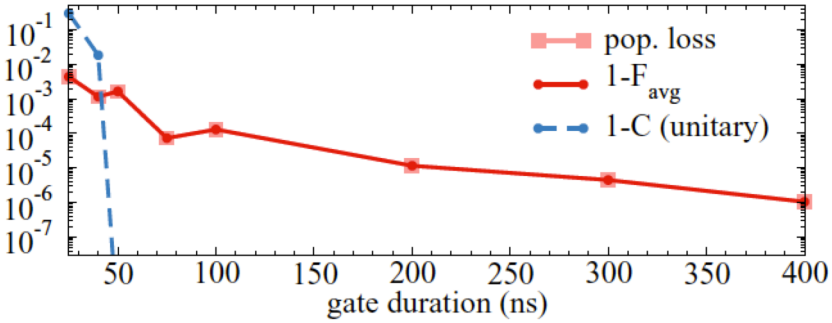
- significant convergence speedup
- example: implementation of B-gate in an effective spin model:

Mueller et al., Phys. Rev. A 84, 042315 (2011)

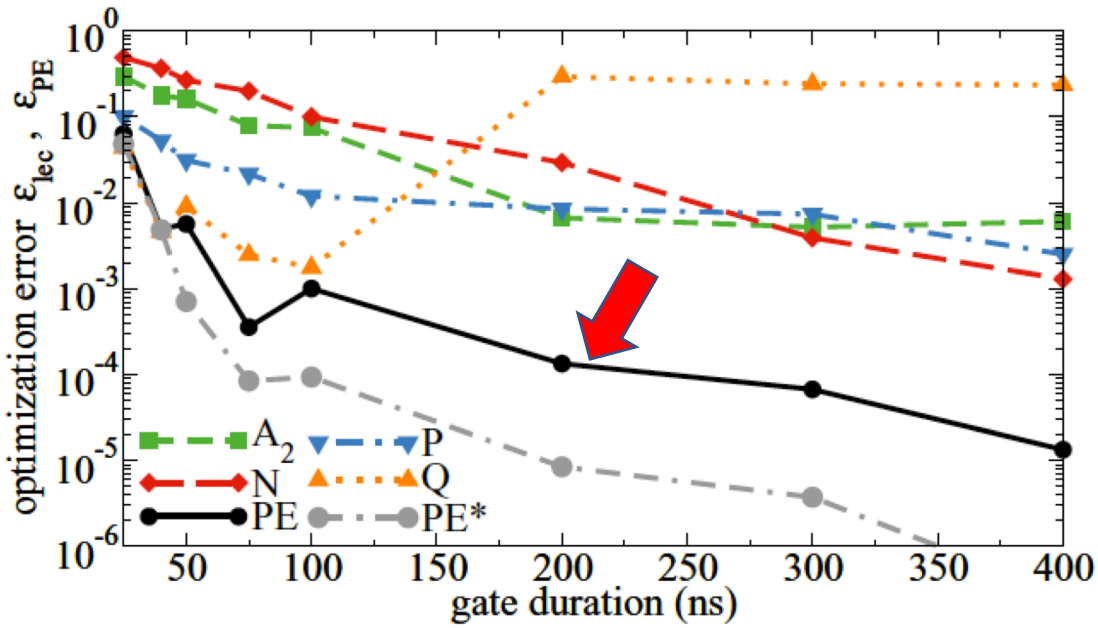


Quantum control to perfect entanglers

- example: transmon qubits



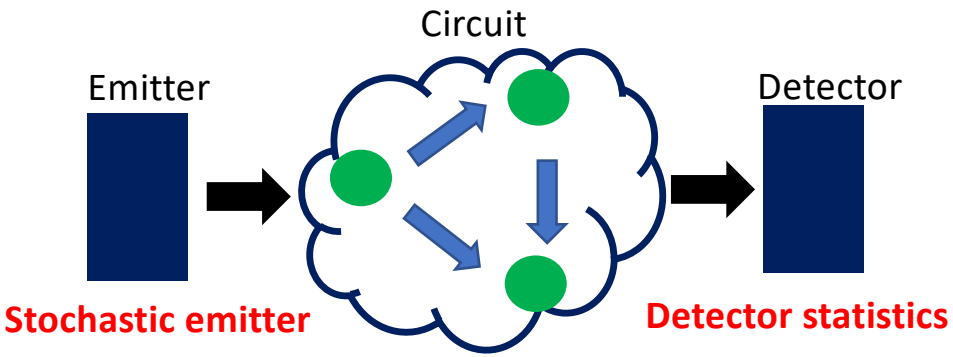
Goerz et al. Phys. Rev. A 91, 062307 (2015)
Watts et al., Phys. Rev. A 91, 062306 (2015)



Photonic Quantum Computing

DTIF Project QColr

SOQCS Stochastic Optical Quantum Circuit Simulator

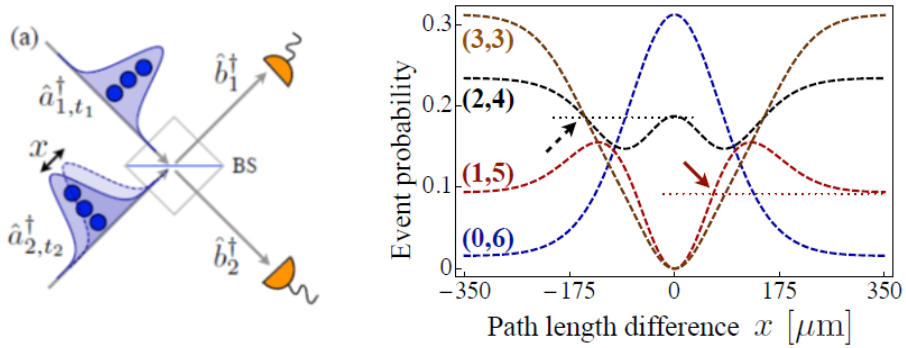


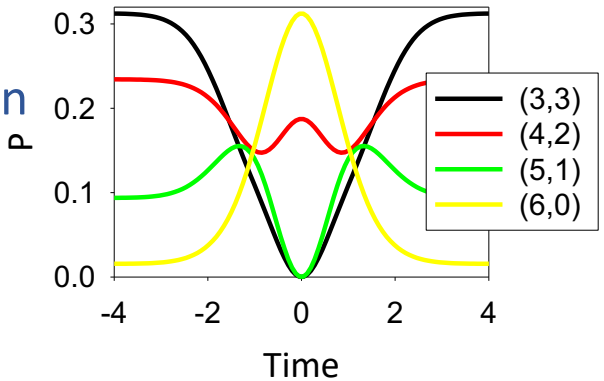
Osca, Vala, publication in preparation (2022).

- efficient numerical encoding of photonic quantum states
- numerical implementation of linear optics elements
- stochastic implementation of imperfect emitters
- reproducing correct detector statistics
- implemented partial photon distinguishability/visibility
- implemented lossy optical elements

Excellent agreement of SOQCS simulations with analytical and published results

- example: beamsplitter with partial photon distinguishability/visibility



Tichy, J. Phys. B: At. Mol. Opt. Phys. 47 103001 (2014)

SOQCS simulation